FACULTAD DE INGENIERIA

Facultad de Ingeniería
Comisión Académica de Posgrado

1007/10

Formulario de Aprobación Curso de Posgrado

Asignatura: An introduction to intuitionistic and classical realizability.

Profesor de la asignatura [1] : Dr. Alexandre Miquel. Maître de Conférences, École Normale Supérieure de Lyon

Profesor Responsable Local [1] : Dr. Marco Scavino (grado 3 del IMERL)

Instituto ó Unidad: Departamento ó Área: Centro de Matemática – Facultad de Ciencias

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Horario y Salón: horario a determinar. Lugar: Centro de Matemática, Facultad de Ciencias, UdelaR

Horas Presenciales: 30 (sumar horas directas de clase – teóricas, prácticas y laboratorio – horas de estudio asistido y de evaluación) Se deberán discriminar las mismas en el ítem Metodología de enseñanza.

Nº de Créditos: 5

Público objetivo y Cupos: Estudiantes de posgrado en matemática o informática e investigadores de estas disciplinas

Objetivos:

Background
The Curry-Howard correspondence Proof theory was deeply renewed in the last fifty years by the discovery of a strong correspondence between proof theoretical concepts and the concepts of functional programming, known as the Curry-Howard correspondence [1, 4, 2]. According to this correspondence, each formal proof can be seen as a functional program (a.k.a. the proofs-as-programs paradigm) that meets a specification given by the proved formula (a.k.a. the formulae-as-types paradigm). The study of this correspondence had major theoretical outputs in logic, such as the discovery of type theory and of linear logic. On the practical side, the Curry-Howard correspondence led to the development of proof assistants such as Coq, MinLog, Agda, Plastic, NuPrl (based on type theory and its variants) while influencing the design of functional programming languages such as SML, Caml or Haskell.

The theory of realizability The aim of this series of lectures is to introduce the theory of realizability [5, 9, 7], a fundamental tool to analyze the computational meaning of proofs through the Curry-Howard correspondence. Basically, realizability generalizes the idea underlying standard model theory by interpreting every formula (of a given formal system) not as a truth value 0 or 1, but as a set of programs – thus giving $2^{\aleph_0}$ different truth values. One of the main applications of realizability is program extraction, by which a formal existence proof of a function satisfying a given specification can be turned into a program that implements the corresponding function. One of the aims of these lectures is to show how to extract such a program effectively, depending on the formal system the source proof is formalized in.

Intuitionistic versus classical realizability Realizability was long limited to intuitionistic logic and to constructive mathematics. In the 90's, Krivine developed a theory of realizability for classical logic [8], using the connection between control operators (such as call/cc) and non constructive reasoning (such as Peirce's law) discovered earlier by Griffin [3]. Thanks to this tool, it is now possible to extend the proofs-as-programs interpretation to classical mathematics (including weak forms of the axiom of choice) while keeping the possibility of extracting programs from proofs. Moreover, recent work showed that classical realizability appears to be strongly connected with Cohen's theory of forcing (that was introduced to prove the independence of the continuum hypothesis), one of the main sources of inspiration of Krivine's work.

Conocimientos previos exigidos: Nociones básicas de calculo lambda tipado, sistemas F o AF2. Correspondencia de Curry-Howard.

Conocimientos previos recomendados: Conocimiento de algún lenguaje de programación funcional. Familiaridad con lenguajes definidos recursivamente e inducción estructural. Nociones elementales de Teoría de la Demostración y Teoría de Modelos.

Metodología de enseñanza: El curso será dictado en idioma inglés. Son 16 horas de clases teóricas, 6 horas de trabajos dirigidos (practico), 8 horas de consulta y un trabajo obligatorio individual, valido por la ganancia del curso y el examen final. Se estima una cantidad de 45 horas en las que el estudiante trabaje de forma independiente para cumplir con la forma de evaluación.

Forma de evaluación: Trabajo obligatorio final.

Temario:
The series of lectures will be naturally organized in two parts. The first part will be devoted to the standard presentation of realizability - i.e. realizability in intuitionistic logic - including:

- Kleene realizability in HA (using λ-calculus and p.c.a.s)
- Formalized realizability and glued realizability
- Kreisel's modified realizability. Practical program extraction
- Realizability in second- and higher-order Heyting arithmetic

The second part of the series of lectures will be devoted to a more advanced topic which is the recent reformulation by Krivine of the theory of realizability in the framework of classical logic, including:

- Krivine realizability. Models induced by classical realizability
- Program extraction using classical realizability
- Interpretation of the dependent axiom of choice
- Extension to higher-order arithmetic. Connections with forcing

Bibliografía:
(titulo del libro-nombre del autor-editorial-ISBN-fecha de edición)

[1] H. B. Curry and R. Feys. *Combinatory Logic, vol. 1.* North-Holland. Amsterdam, 1958.
[2] J.-Y. Girard, Y. Lafont and P. Taylor. *Proofs and Types.* Cambridge University Press, ISBN: 0521371813, 1989.
[3] T. Griffin. A Formulae-as-Types Notion of Control. *Principles Of Programming Languages (POPL'90)*, 47-58. 1990.
[4] W. A. Howard. *The formulae-as-types notion of construction*, Privately circulated notes. 1969.
[5] S. C. Kleene, On the interpretation of intuitionistic number theory. *Journal of Symbolic Logic*, 10:109-124. 1945.
[6] G. Kreisel. Interpretation of analysis by means of functionals of finite type. In A. Heyting, editor, *Constructivity in Mathematics*, 101-128. North-Holland, 1959.
[7] J.-L. Krivine, *Lambda-calculus, types and models.* Masson, ISBN: 0130624071, 1993.
[8] J.-L. Krivine, Realizability in classical logic. In Interactive models of computation and program behaviour, Panoramas et synthèses, 27:197-229. Société Mathématique de France, 2009.
[9] A. S. Troelstra. *Metamathematical Investigations of Intuitionistic Arithmetic and Analysis.* Lecture Notes in Mathematics 344, Springer, ISBN: 0387064914, 1973. With contributions of A. S. Troelstra, C. A. Smoryński, J. I. Zucker and W.A. Howard.